

*Mastery Exams in Calculus I*  
*A Preliminary Report*

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# Overview

**The History of Mastery Exams**

**Description of Mastery Exams**

**The Impact of Mastery Exams**

**The Future of Mastery Exams**

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- Questions which test *only* these skills are removed from quizzes and exams.

# The History of Mastery Exams

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- technical mastery became a stated goal of the course
- Mastery Exams achieve this, make room in evaluation tools for problem-solving, abstraction, proof

# First Incarnation

## Kalamazoo College

- ▶ Algebra
- ▶ Derivatives
- ▶ Calculus II Skills (integrals, series)
- ▶ Calculus III Skills (equation of tangent plane, Lagrange multipliers, double integrals, etc.)

# Adaptation

Washington & Jefferson College

- ▶ Calculus I, Fall 2008 (3 sections)
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- ▶ Limits
- ▶ Derivatives

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Sample topics:

1. factoring
2. change absolute value to piecewise-defined function
3. solve inequalities
4. evaluate function at expression
5. point-slope form of a line

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3. Write the equation of the line which passes through the point  $(-2, 3)$  and is parallel to the line  $-x + 3y + 2 = 0$ .

# The Limits Mastery Exam

Sample questions:

1. Evaluate the following limit, if it exists. If it does not exist, explain why in detail.

(a)  $\lim_{w \rightarrow 9} \frac{-\frac{2}{w} + \frac{4}{18}}{-w^2 + 81}$

(b)  $\lim_{u \rightarrow 8} q(u)$  where  $q(u) = \begin{cases} u^2 - 4u - 32 & u < 8 \\ 2u^2 - 18u + 16 & u > 8 \end{cases}$

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2. Determine if the following function has any vertical asymptotes. If it does, describe the behavior of the function (using limit notation) near each vertical asymptote.

$$w(y) = \frac{y^2 - 11y + 24}{y^2 - 10y - 24}$$

# The Derivative Mastery Exam

Sample questions:

1. Find the derivatives of the following functions.

$$(a) h(t) = \frac{\sin(t) + 4}{5t^8 - 2t}$$

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2. Find  $\frac{dy}{dx}$  if  $x$  and  $y$  are related by the following equation.

$$xy - xy^2 = 3x + 7 - y^3$$

# The Impact of Mastery Exams

# The Effects on Grades

## Exam #1

- worth a smaller percentage of overall grade
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## Overall Grades

- ▶ virtually the same mean and median at the end of the course (as compared to traditional course)

# Exam 1 Problems

More interesting problems on Exam #1:

An airplane is traveling in a path shaped like the graph of the parabolic function  $f(x) = x^2 - x$ . The airplane starts at a point 5 miles west and 30 miles north of the origin, traveling east along this path. At what point will the airplane be flying directly toward the point  $(-1, 0)$ ?

# Student Response

## Student Evaluations:

- Algebra: 5.6/6.0      Limits: 5.4/6.0      Derivatives: 5.5/6.0

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- Algebra: 5.6/6.0      Limits: 5.4/6.0      Derivatives: 5.5/6.0
- “I liked how the instructor made me explain every answer each time.”
- “The mastery exams helped me grasp all the concepts and improved my grade. I would not change anything about them.”
- “They were the best part of the class.”
- “Have more of them.”
- 1 negative comment between 3 sections in Fall 2008

# Faculty Response

## Positive

- personalized tutorial for students; students visit office early in semester
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## Negative

- time, time, time
- students may compartmentalize these questions/skills
- student performance/morale surrounding first exam

# The Future of Mastery Exams

# What's Next?

- Study the effectiveness of Mastery Exams during the 2009–2010 academic year.

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- Study the effectiveness of Mastery Exams during the 2009–2010 academic year.
- Implement Mastery Exams in Calculus II, Fall 2010.