The Nilpotent Filtration in Group Cohomology

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Abstract

Let P be a finite p-group and let e be an idempotent in $\mathbb{F}_p[\mathrm{Out}(P)]$. In this dissertation we explore the Krull dimension of $eH^*(P;\mathbb{F}_p)$. It is known that this dimension cannot exceed the largest rank of an elementary abelian p-subgroup of P. We investigate conditions on P which ensure that $\dim(eH^*(P))$ is maximal.

The nilpotent filtration of the category of unstable modules over the Steenrod algebra plays a key role in the solutions we present. In particular, the dimension of a module depends only on the size of the subquotients in its nilpotent filtration. We also rely on the descriptions of the localization of $\mathcal{U}$ with respect to the categories $\mathcal{N}il_n$ given by H. W. Henn, J. Lannes, and L. Schwartz in [19] and [20].

Our main results come in the form of two separate group theoretic criteria. For a group P, $\dim(eH^*(P))$ is maximal if:

- P has an elementary abelian p-subgroup of maximal rank which is both normal in P and self-centralizing; or
- all elements of order p are central.

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"Here I raise my Ebenezer,

Hither by Thy help I'm come."

Soli Deo Gloria